1. The lifetime, X, in tens of hours, of a battery has a cumulative distribution function F(x) given by

$$\mathrm{F}(x)=\begin{cases}0 & x<1\\ \frac{4}{9}\left(x^2+2x-3\right) & 1\le x\le 1.5\\ 1 & x>1.5\end{cases}$$

(a) Find the median of X, giving your answer to 3 significant figures.

(3)

(b) Find, in full, the probability density function of the random variable X.

(3)

(c) Find P($X \geq 1.2$)

(2)

A camping lantern runs on 4 batteries, all of which must be working. Four new batteries are put into the lantern.

(d) Find the probability that the lantern will still be working after 12 hours.

(2)
(Total 10 marks)

2. The random variable y has probability density function f(y) given by

$$\mathrm{f}(y)=\begin{cases}ky(a-y) & 0\le y\le 3\\ 0 & \text{otherwise}\end{cases}$$

where k and a are positive constants.

(a) (i) Explain why $a \geq 3$

(ii) Show that $k=\dfrac{2}{9(a-2)}$

(6)

Given that E(Y) = 1.75

(b) show that a = 4 and write down the value of k.

(6)

For these values of a and k,

(c) sketch the probability density function,

(2)

(d) write down the mode of Y.

(1)
(Total 15 marks)

3. A continuous random variable x has cumulative distribution function

$$\mathrm{F}(x) = \begin{cases} 0, & x < -2 \\ \dfrac{x+2}{6}, & -2 \le x \le 4 \\ 1, & x > 4 \end{cases}$$

(a) Find P(X < 0).

(2)

(b) Find the probability density function f(x) of X.

(3)

(c) Write down the name of the distribution of X.

(1)

(d) Find the mean and the variance of X.

(3)

(e) Write down the value of P(X = 1).

(1)
(Total 10 marks)

4. The continuous random variable x has probability density function f(x) given by

$$f(x) = \begin{cases} k(x^2 - 2x + 2) & 0 < x \leq 3 \\ 3k & 3 < x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(a) Show that $k = \dfrac{1}{9}$

(4)

(b) Find the cumulative distribution function F(x).

(6)

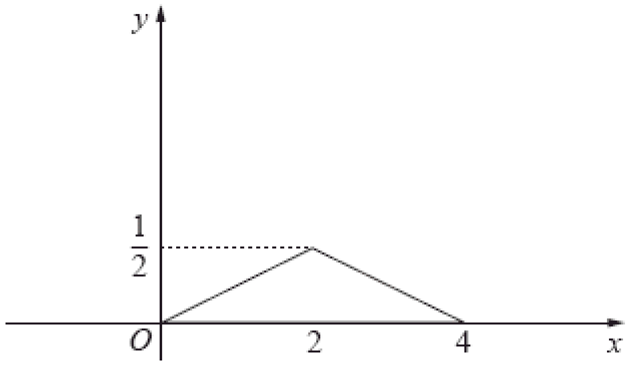
(c) Find the mean of X.

(3)

(d) Show that the median of X lies between x = 2.6 and x = 2.7

(4)
(Total 17 marks)

5.



The diagram above shows a sketch of the probability density function f(x) of the random variable X. The part of the sketch from $x = 0$ to $x = 4$ consists of an isosceles triangle with maximum at (2, 0.5).

(a) Write down E(X).

(1)

The probability density function f(x) can be written in the following form.

$$\mathrm{f}(x) = \begin{cases} ax & 0 \le x \le 2 \\ b - ax & 2 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

(b) Find the values of the constants a and b.

(2)

(c) Show that σ, the standard deviation of X, is 0.816 to 3 decimal places.

(7)

(d) Find the lower quartile of X.

(3)

(e) State, giving a reason, whether P($2 - \sigma < X < 2 + \sigma$) is more or less than 0.5

(2)
(Total 15 marks)

6. The length of a telephone call made to a company is denoted by the continuous random variable T. It is modelled by the probability density function

$$\mathrm{f}(t) = \begin{cases} kt & 0 \le t \le 10 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the value of k is $\frac{1}{50}$.

(3)

(b) Find P(T > 6).

(2)

(c) Calculate an exact value for E(T) and for Var(T).

(5)

(d) Write down the mode of the distribution of T.

(1)

It is suggested that the probability density function, f(t), is not a good model for T.

(e) Sketch the graph of a more suitable probability density function for T.

(1)
(Total 12 marks)

7. A random variable X has probability density function given by

$$\mathrm{f}(x) = \begin{cases} -\frac{2}{9}x + \frac{8}{9} & 1 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the cumulative distribution function F(x) can be written in the form $ax^2 + bx + c$, for $1 \le x \le 4$ where a, b and c are constants.

(3)

(b) Define fully the cumulative distribution function F(x).

(2)

(c) Show that the upper quartile of X is 2.5 and find the lower quartile.

(6)

Given that the median of X is 1.88

(d) describe the skewness of the distribution. Give a reason for your answer.

(2)
(Total 13 marks)

8. A random variable X has probability density function given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{2}x & 0 \le x \le 1 \\ kx^3 & 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{1}{5}$

(4)

(b) Calculate the mean of X.

(4)

(c) Specify fully the cumulative distribution function $\mathrm{F}(x)$.

(7)

(d) Find the median of X.

(3)

(e) Comment on the skewness of the distribution of X.

(2)
(Total 20 marks)

9. The continuous random variable Y has cumulative distribution function F(y) given by

$$\mathrm{F}(y) = \begin{cases} 0 & y < 1 \\ k(y^4 + y^2 - 2) & 1 \le y \le 2 \\ 1 & y > 2 \end{cases}$$

(a) Show that $k = \dfrac{1}{18}$.

(2)

(b) Find P(Y > 1.5).

(2)

(c) Specify fully the probability density function f(y).

(3)
(Total 7 marks)

10. The continuous random variable X has probability density function f(x) given by

$$\mathrm{f}(x) = \begin{cases} 2(x-2) & 2 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch f(x) for all values of x.

(3)

(b) Write down the mode of X.

(1)

Find

(c) E(X),

(3)

(d) the median of X.

(4)

(e) Comment on the skewness of this distribution. Give a reason for your answer.

(2)
(Total 13 marks)

11. The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{6}x & 0 < x \leq 3 \\ 2 - \frac{1}{2}x & 3 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the probability density function of X.

(3)

(b) Find the mode of X.

(1)

(c) Specify fully the cumulative distribution function of X.

(7)

(d) Using your answer to part (c), find the median of X.

(3)
(Total 14 marks)

12. The continuous random variable X is uniformly distributed over the interval $\alpha < x < \beta$.

(a) Write down the probability density function of X, for all x.

(2)

(b) Given that E(X) = 2 and P(X < 3) = $\frac{5}{8}$ find the value of α and the value of β.

(4)

A gardener has wire cutters and a piece of wire 150 cm long which has a ring attached at one end. The gardener cuts the wire, at a randomly chosen point, into 2 pieces. The length, in cm, of the piece of wire with the ring on it is represented by the random variable X. Find

(c) E(X),
(1)

(d) the standard deviation of X,
(2)

(e) the probability that the shorter piece of wire is at most 30 cm long.
(3)
(Total 12 marks)

13. The continuous random variable X has cumulative distribution function

$$\mathrm{F(x)} = \begin{cases} 0, & x < 0 \\ 2x^2 - x^3, & 0 \le x \le 1 \\ 1, & x > 1. \end{cases}$$

(a) Find P(X > 0.3).
(2)

(b) Verify that the median value of X lies between x = 0.59 and x = 0.60.
(3)

(c) Find the probability density function f(x).
(2)

(d) Evaluate E(X).
(3)

(e) Find the mode of X.
(2)

(f) Comment on the skewness of X. Justify your answer.

(2)
(Total 14 marks)

14. The continuous random variable X has probability density function

$$f(x) = \begin{cases} \dfrac{1+x}{k}, & 1 \le x \le 4 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Show that $k = \dfrac{21}{2}$.

(3)

(b) Specify fully the cumulative distribution function of X.

(5)

(c) Calculate $E(X)$.

(3)

(d) Find the value of the median.

(3)

(e) Write down the mode.

(1)

(f) Explain why the distribution is negatively skewed.

(1)
(Total 16 marks)

15. A continuous random variable X has probability density function f(x) where

$$\mathrm{f}(x)=\begin{cases} kx(x-2), & 2 \le x \le 3, \\ \\ 0, & \text{Otherwise} \end{cases}$$

where k is a positive constant.

(a) Show that $k=\frac{3}{4}$.

(4)

Find

(b) E(X),

(3)

(c) the cumulative distribution function F(x).

(6)

(d) Show that the median value of X lies between 2.70 and 2.75.

(2)
(Total 15 marks)

16. A continuous random variable X has probability density function f(x) where

$$\mathrm{f}(x) = \begin{cases} k(4x-x^3), & 0 \le x \le 2, \\ 0, & \text{otherwise,} \end{cases}$$

where k is a positive constant.

(a) Show that $k = \frac{1}{4}$.

(4)

Find

(b) E(X),

(3)

(c) the mode of X,

(3)

(d) the median of X.

(4)

(e) Comment on the skewness of the distribution.

(2)

(f) Sketch f(x).

(2)
(Total 18 marks)

17. The random variable X has probability density function

$$f(x) = \begin{cases} k(-x^2 + 5x - 4), & 1 \le x \le 4, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Show that $k = \frac{2}{9}$.

(3)

Find

(b) E(X),

(3)

(c) the mode of X.

(2)

(d) the cumulative distribution function $F(x)$ for all x.

(5)

(e) Evaluate $P(X \leq 2.5)$.

(2)

(f) Deduce the value of the median and comment on the shape of the distribution.

(2)
(Total 17 marks)

18. A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{3}, & 0 \leq x \leq 1, \\ \frac{8x^3}{45}, & 1 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Calculate the mean of X.

(5)

(b) Specify fully the cumulative distribution function $F(x)$.

(7)

(c) Find the median of X.

(3)

(d) Comment on the skewness of the distribution of X.

(2)
(Total 17 marks)

19. The continuous random variable X has probability density function

$$f(x) = \begin{cases} kx(5-x), & 0 \le x \le 4 \\ 0, & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{3}{56}$.

(3)

(b) Find the cumulative distribution function F(x) for all values of x.

(4)

(c) Evaluate E(X).

(3)

(d) Find the modal value of X.

(3)

(e) Verify that the median value of X lies between 2.3 and 2.5.

(3)

(f) Comment on the skewness of X. Justify your answer.

(2)
(Total 18 marks)

20. A continuous random variable X has probability density function f(x) where

$$f(x) = \begin{cases} k(x^2 + 2x + 1) & -1 \le x \le 0, \\ 0, & \text{otherwise} \end{cases}$$

where k is a positive integer.

(a) Show that $k = 3$.

(4)

Find

(b) E(X),

(4)

(c) the cumulative distribution function F(x),

(4)

(d) P($-0.3 < X < 0.3$).

(3)
(Total 15 marks)

21. The continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0, \\ \frac{1}{3}x^2(4 - x^2), & 0 \le x \le 1, \\ 1, & x > 1. \end{cases}$$

(a) Find P($X > 0.7$).

(2)

(b) Find the probability density function f(x) of X.

(2)

(c) Calculate E(X) and show that, to 3 decimal places, Var (X) = 0.057.

(6)

One measure of skewness is

$$\frac{\text{Mean} - \text{Mode}}{\text{Standard deviation}}.$$

(d) Evaluate the skewness of the distribution of X.

(4)
(Total 14 marks)

22. The lifetime, in tens of hours, of a certain delicate electrical component can be modelled by the random variable X with probability density function

$$f(x) = \begin{cases} \frac{1}{42}x, & 0 \le x < 6, \\ \frac{1}{7} & 6 \le x \le 10, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Sketch f(x) for all values of x.

(4)

(b) Find the probability that a component lasts at least 50 hours.

(3)

A particular device requires two of these components and it will not operate if one or more of the components fail. The device has just been fitted with two new components and the lifetimes of these two components are independent.

(c) Find the probability that the device breaks down within the next 50 hours.

(2)
(Total 9 marks)

23. The continuous random variable T represents the time in hours that students spend on homework. The cumulative distribution function of T is

$$F(t) = \begin{cases} 0, & t < 0, \\ k(2t^3 - t^4) & 0 \le t \le 1.5, \\ 1, & t > 1.5. \end{cases}$$

where k is a positive constant.

(a) Show that $k = \frac{16}{27}$.

(2)

(b) Find the proportion of students who spend more than 1 hour on homework.

(2)

(c) Find the probability density function f(t) of T.

(3)

(d) Show that $\mathrm{E}(T) = 0.9$.

(3)

(e) Show that $\mathrm{F}(\mathrm{E}(T)) = 0.4752$.

(1)

A student is selected at random. Given that the student spent more than the mean amount of time on homework,

(f) find the probability that this student spent more than 1 hour on homework.

(3)
(Total 14 marks)

1. (a) $\frac{4}{9}\left(m^2+2m-3\right)$ = 0.5 M1

$m^2 + 2m - 4.125 = 0$

$m = \dfrac{-2 \pm \sqrt{4+16.5}}{2}$ M1

m =1.26, –3.264

(median =) 1.26 A1 3

Note

M1 putting F(x) = 0.5
M1 using correct quadratic formula. If use calc need to get 1.26 (384...)
A1 cao 1.26 must reject the other root.
If they use Trial and improvement they have to get the correct answer to gain the second M mark.

(b) Differentiating $\dfrac{\mathrm{d}\left(\frac{4}{9}\left(x^2+2x-3\right)\right)}{\mathrm{d}x} = \frac{4}{9}\left(2x+2\right)$ M1 A1

$$\mathrm{f}(x) = \begin{cases} \frac{8}{9}\left(x+1\right) & 1 \le x \le 1.5 \\ 0 & \text{otherwise} \end{cases}$$ B1ft 3

Note

M1 attempt to differentiate. At least one $x^n \to x^{n-1}$
A1 correct differentiation
B1 must have both parts- follow through their F′(x) Condone <

(c) P($X \ge 1.2$) = 1 – F(1.2) M1
= 1 – 0.3733
$= \frac{47}{75}$, 0.6267 awrt 0.627 A1 2

0.627

Note

M1 finding/writing 1 – F(1.2) may use/write $\int_{1.2}^{1.5}\frac{8}{9}\left(x+1\right)\mathrm{d}x$

or 1 – $\int_{1}^{1.2}\frac{8}{9}\left(x+1\right)\mathrm{d}x$ or $\int_{1.2}^{1.5}$"their f (x)"dx .Condone missing dx

A1 awrt 0.627

(d) $(0.6267)^4 = 0.154$ awrt 0.154 or 0.155 M1 A1 2

Note

M1 $(\mathrm{c})^4$ If expressions are not given you need to check the calculation

is correct to 2sf.
A1 awrt 0.154 or 0.155

[10]

2. (a) (i) f(y) ≥ 0 or f(3) ≥ 0 M1

$ky(a-y) \geq 0$ or $3k(a-3) \geq 0$ or $(a-y) \geq 0$ or $(a-3) \geq 0$

$a \geq 3$ A1 cso

Note

M1 for putting f(y) ≥ 0 or f(3) ≥ 0 or $ky(a-y) \geq 0$ or $3k(a-3) \geq 0$ or $(a-y) \geq 0$ or $(a-3) \geq 0$ or state in words the probability can not be negative o.e.
A1 need one of $ky(a-y) \geq 0$ or $3k(a-3) \geq 0$ or $(a-y) \geq 0$ or $(a-3) \geq 0$ **and** $a \geq 3$

(ii) $\int_0^3 k(ay-y^2)dy = 1$ integration M1

$\left[k\left(\frac{ay^2}{2}-\frac{y^3}{3}\right)\right]_0^3 = 1$ answer correct A1

$k\left(\frac{9a}{2}-9\right) = 1$ answer = 1 M1

$k\left[\frac{9a-18}{2}\right] = 1$

$k = \frac{2}{9(a-2)}$ * A1 cso 6

Note

M1 attempting to integrate (at least one $y^n \to y^{n+1}$) (ignore limits)
A1 Correct integration. Limits not needed. And equals 1 not needed.
M1 dependent on the previous M being awarded. Putting equal to 1 and have the correct limits. Limits do not need to be substituted.
A1 cso

(b) $\int_0^3 k(ay^2-y^3)\mathrm{dy} = 1.75$ Int $\int xf(x)$ M1

$\left[k\left(\frac{ay^3}{3}-\frac{y^4}{4}\right)\right]_0^3 = 1.75$

Correct integration A1

$\int xf(x) = 1.75$ and limits 0,3 M1dep

$k\left(9a - \frac{81}{4}\right) = 1.75$

$2\left(9a - \frac{81}{4}\right) = 15.75(a-2)$ subst k M1dep

$2.25a = -31.5 + \frac{81}{2}$

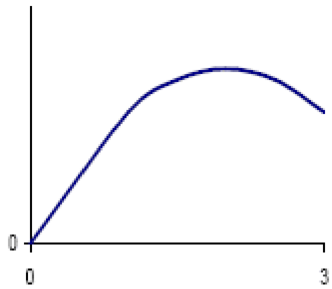
a = 4 * A1cso

$k = \frac{1}{9}$ B1 6

Note

M1 for attempting to find $\int y\text{f}(y)\,\text{d}y$ (at least one $y^n \to y^{n+1}$)
(ignore limits)
A1 correct Integration
M1 $\int y\text{f}(y) = 1.75$ and limits 0, 3 dependent on previous M being awarded
M1 subst in for k. dependent on previous M being awarded
A1 cso 4
B1 cao 1/9

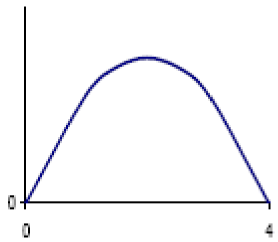
(c)



B1
B1 2

Note

B1 correct shape. No straight lines. No need for patios.
B1 completely correct graph. Needs to go through origin and the curve ends at 3.
Special case: If draw full parabola from 0 to 4 get B1 B0 Allow full marks if the portion between x = 3 and x = 4 is dotted and the rest of the curve solid.



(d) mode = 2 B1 1

Note

B1 cao 2

[15]

3. (a) P(X < 0) = F(0) M1

$= \frac{2}{6} = \frac{1}{3}$ A1 2

Note

M1 for attempting to find F(0) by a correct method

eg subst 0 into F(x) or $\int_{-2}^{0} \frac{1}{6} dx$

Do **NOT** award M1 for $\int_{-2}^{0} \frac{x+2}{6} dx$ or $\frac{1}{2} \times \frac{1}{3} \times 2$

both of which give the correct answer by using F(x) as the pdf

A1 1/3 o.e or awrt 0.333

Correct answer only with no incorrect working gets M1 A1

(b) f(x) = $\frac{\mathrm{dF}(x)}{\mathrm{d}x}$ M1

$$\mathrm{f}(x) = \begin{cases} \frac{1}{6} & -2 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

A1

B1 3

Note

M1 for attempting to differentiate F(x).
(for attempt it must have no xs in)

A1 for the first line. Condone < signs

B1 for the second line. – They must have
0 x < –2 and x < 4 only.

(c) Continuous Uniform (Rectangular) distribution B1 1

Note

B1 must have "continuous" and "uniform" **or** "Rectangular"

(d) Mean = 1 B1

Variance is $\frac{(4--2)^2}{12} = 3$ M1 A1 3

Note

B1 for mean = 1

M1 for attempt to use $\frac{[\pm(b-a)]^2}{12}$, they must subst in values and not just quote the formula, or using $\int_{-2}^{4} x^2$ (*their f*(*x*)) – (*their mean*)^2, including limits. Must get x^3

when they integrate.

A1 cao .

(e) P(X = 1) = 0 B1 1

Note

B1 cao

[10]

4. (a) $\int_0^3 k(x^2 - 2x + 2)\mathrm{d}x + \int_3^4 3k\mathrm{d}x = 1$ M1

$k\left[\frac{1}{3}x^3 - x^2 + 2x\right]_0^3 + [3kx]_3^4 \ (=1)$ or

$k\left[\frac{1}{3}x^3 - x^2 + 2x\right]_0^3 + 3k \ (=1)$ A1

$9k = 1$ M1 dep

$k = \frac{1}{9}$ * * given * * cso A1 4

Note

1st M1 attempting to integrate **at least one** part (at least one $x^n \to x^{n+1}$) (ignore limits)

1st A1 Correct integration. Limits not needed.

2nd M1 dependent on the previous M being awarded. Adding the two answers together, putting equal to 1 and have the correct limits.

2nd A1 cso

(b) For $0 > x \leq 3$, F(x)	$= \int_0^x \frac{1}{9}(t^2 - 2t + 2)\mathrm{d}t$	M1	
	$= \frac{1}{9}\left(\frac{1}{3}x^3 - x^2 + 2x\right)$	A1	
For $3 < x \leq 4$, F(x)	$= \int_3^x 3k\mathrm{d}t + \frac{2}{3}$	M1	
	$= \frac{x}{3} - \frac{1}{3}$	A1	
F(x)	$= \begin{cases} 0 & x \leq 0 \\ \frac{1}{27}(x^3 - 3x^2 + 6x) & 0 < x \leq 3 \\ \frac{x}{3} - \frac{1}{3} & 3 < x \leq 4 \\ 1 & x > 4 \end{cases}$	B1 ft	
		B1	6

Note

1st M1 Att to integrate $\frac{1}{9}(t^2 - 2t + 2)$ (at least one $x^n \to x^{n+1}$). Ignore limits for method mark

1st A1 $\frac{1}{9}\left(\frac{x^3}{3} - x^2 + 2x\right)$ allow use of t.

Must have used/implied use of limit of 0.

This must be on its own without anything else added

2nd M1 attempting to find $\int_3^x 3k + ...$ (must get $3kt$ or $3kx$)

and they must use the correct limits and add $\int_0^3 \frac{1}{9}(t^2 - 2t + 2)$or $\frac{2}{3}$ or use + C and use F(4) = 1

2nd A1 $\frac{x}{3} - \frac{1}{3}$ must be correct

1st B1 middle pair followed through from their answers. condone them using < or < incorrectly they do not need to match up

2nd B1 end pairs. condone them using < or $\leq$. They do not need to match up

NB if they show no working and just write down the distribution. If it is correct they get full marks. If it is incorrect then they cannot get marks for any incorrect part. So if $0<x\le 3$ is correct they can get M1 A1 otherwise M0 A0. If $3<x\le 4$ is correct they can get M1 A1 otherwise M0 A0. you cannot award B1ft if they show no working unless the middle parts are correct.

(c)	$\text{E}(X) = \int_0^3 \frac{x}{9}\left(x^2 - 2x + 2\right)\text{d}t + \int_3^4 \frac{x}{3}\text{d}x$	M1	
	$= \frac{1}{9}\left[\frac{1}{4}x^4 - \frac{2}{3}x^3 + x^2\right]_0^3 + \left[\frac{1}{6}x^2\right]_3^4$	A1	
	$= \frac{29}{12}$ or 2.416 or awrt 2.42	A1	3

Note

1st M1 attempting to use integral of x f(x) on one part

1st A1 Correct Integration for both parts added together. Ignore limits.

2nd A1 cao or awrt 2.42

(d)	F(m) = 0.5	M1	
	$\text{F}(2.6) = \frac{1}{27}\left(2.6^3 - 3\times 2.6^2 + 6 + 2.6\right) = \text{awrt}\, 0.48$	M1	
	$\text{F}(2.7) = \frac{1}{27}\left(2.7^3 - 3\times 2.7^2 + 6 + 2.7\right) = \text{awrt}\, 0.52$	A1	
	Hence median lies between 2.6 and 2.7	A1 dA	4

Note

1st M1 for using F(X) = 0.5. This may be implied by subst into F(X) and comparing answers with 0.5.

2nd M1 for substituting both 2.6 and 2.7 into "their F(X)" – 0.5 or "their F(X)"

1st A1 awrt 0.48 and 0.52 if using "their F(X)" and awrt –0.02 and 0.02 or if using "their F(X)" 0.5

Other values possible. You may need to check their values for their **correct** equation **NB these last two marks are B1 B1 on ePEN but mark as M1 A1**

2nd A1 for conclusion but only award if it follows from their numbers. Dependent on previous A mark being awarded

SC using calculators

M1 for sign of a suitable equation

M1 A1 for awrt 2.66 provided equation is correct

A1 correct comment

[17]

5.	(a)	$E(X) = 2$ (by symmetry)	B1	1
		Note		
		B1 cao		
	(b)	$0 \le x < 2$, gradient $= \frac{\frac{1}{2}}{2} = \frac{1}{4}$ and equation is		
		$y = \frac{1}{4}x$ so $a = \frac{1}{4}$	B1	
		$b - \frac{1}{4}x$ passes through (4, 0) so $b = 1$	B1	2
		Note		
		B1 for value of a. B1 for value of *b*		
	(c)	$E(X^2) = \int_0^2 \left(\frac{1}{4}x^3\right) dx + \int_2^4 \left(x^2 - \frac{1}{4}x^3\right) dx$	M1M1	
		$= \left[\frac{x^4}{16}\right]_0^2 + \left[\frac{x^3}{3} - \frac{x^4}{16}\right]_2^4$	A1	
		$= 1 + \frac{64-8}{3} - \frac{256-16}{16} \quad = \quad 4\frac{2}{3}$ or $\frac{14}{3}$	M1A1	
		$\text{Var}(X) = E(X^2) - [E(X)]^2 = \frac{14}{3} - 2^2$, $= \frac{2}{3}$		
		$\left(\text{so } \sigma = \sqrt{\frac{2}{3}} = 0.816\right)$ (*)	M1	
			A1cso	7

Note

1st M1 for attempt at $\int ax^3$ using their *a*. For attempt they need x^4. Ignore limits.

2nd M1 for attempt at $\int bx^2 - ax^3$ use their *a* and *b*. For attempt need to have either x^3 or x^4. Ignore limits

1st A1 correct integration for both parts

3rd M1 for use of the correct limits on each part

2nd A1 for either getting 1 and $3\frac{2}{3}$ or awrt 3.67 somewhere or $4\frac{2}{3}$ or awrt 4.67

4th M1 for use of $E(X^2)–[E(X)]^2$ must add both parts for E (X^2) and only have subtracted the mean^2 once. You must see this working

3rd A1 $\sigma = \sqrt{\frac{2}{3}}$ or $\sqrt{0.66667}$ or better with no incorrect working seen.

(d) $\mathrm{P}(X \le q) = \int_0^q \frac{1}{4}x\,\mathrm{d}x = \frac{1}{4}$,

$\frac{q^2}{2} = 1$ so $q = \sqrt{2} = 1.414$ awrt 1.41 M1A1A1 3

Note

M1 for attempting to find LQ, integral of either part of f(x) with their 'a' and 'b' = 0.25

Or their F(x) = 0.25 i.e. $\frac{ax^2}{2}$ = 0.25 or

$bx - \frac{ax^2}{2} + 4a - 2b = 0.25$ with their a and b

If they add both parts of their F(x), then they will get M0.

1st A1 for a correct equation/expression using their 'a'

(e) 2– σ = 1.184 so 2 – σ, 2 + σ is wider than IQR, therefore greater than 0.5 M1 A1 2

Note

2nd A1 for $\sqrt{2}$ or awrt 1.41

M1 for a reason based on their quartiles

- Possible reasons are P(2 – σ < X < 2 + σ) = 0.6498 allow awrt 0.65
- 1.184 < LQ(1.414)

A1 for correct answer > 0.5

NB you must check the reason and award the method mark. A correct answer without a correct reason gets M0 A0

[15]

6. (a) $\int_0^{10} kt\,\mathrm{d}t = 1$ or Area of triangle = 1 M1

$\left[\frac{kt^2}{2}\right]_0^{10} = 1$ or 10 × 0.5 × 10k =1 or linear equation in k M1

$50k = 1$

$k = \frac{1}{50}$ cso A1 3

(b) $\int_0^{10} kt\text{d}t = \left[\frac{\text{kt}^2}{2}\right]_6^{10}$ M1

$= \frac{16}{25}$ A1 2

(c) E(T) = $\int_0^{10} kt^2\text{d}t = \left[\frac{\text{kt}^3}{3}\right]_0^{10}$ M1

$= 6\frac{2}{3}$ A1

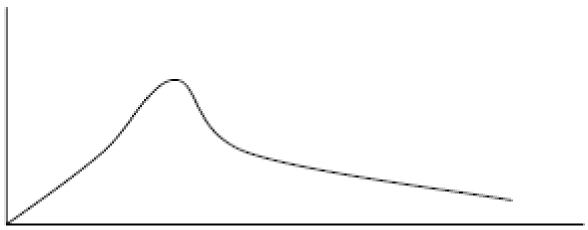
Var (T) = $\int_0^{10} kt^3\text{d}t - \left(6\frac{2}{3}\right)^2 = \left[\frac{kt4}{4}\right]_0^{10}; - \left(6\frac{2}{3}\right)^2$ M1; M1dep

$= 50 - \left(6\frac{2}{3}\right)^2$

$= 5\frac{5}{9}$ A1 5

(d) 10 B1 1

(e)



B1 1

[12]

7 (a) F(x_0) = $\int_1^x -\frac{2}{9}x + \frac{8}{9}\text{dx} = \left[-\frac{1}{9}x^2 + \frac{8}{9}x\right]_1^x$ M1A1

$= \left[-\frac{1}{9}x^2 + \frac{8}{9}x\right] - \left[-\frac{1}{9} + \frac{8}{9}\right]$

$= -\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9}$ A1 3

(b) F(x) = $\begin{cases} 0 & x < 1 \\ -\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} & 1 \le x \le 4 \\ 1 & x > 4 \end{cases}$ B1B1ft 2

(c) F(x) = 0.75; or $F(2.5) = -\frac{1}{9}\times 2.5^2 + \frac{8}{9}\times 2.5 - \frac{7}{9}$ M1;

$-\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} = 0.75$

$4x^2 - 32^x + 55 = 0$

$-x^2 + 8x - 13.75 = 0$

$x = 2.5$ $= 0.75$ cso A1

and F(x) = 0.25

$-\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} = 0.25$ M1

$-x^2 + 8x - 7 = 2.25$

$-x^2 + 8x - 9.25 = 0$ quadratic 3 terms =0 M1 dep

$x = \dfrac{-8 \pm \sqrt{8^2 - 4\times -1\times -9.25}}{2\times -1}$ M1 dep

$x = 1.40$ A1 6

(d) $Q_3 - Q_2 > Q_2 - Q_1$ M1

Or mode = 1 and mode < median

Or mean = 2 and median < mode

Sketch of pdf here or be referred to if in a different part of the question

Box plot with Q_1, Q_2, Q_3 values marked on

Positive skew A1 2

[13]

8. (a) $\int_0^1 \frac{1}{2}x\,dx = \left[\frac{1}{4}x^2\right]_0^1 = \frac{1}{4}$ oe attempt to integrate both parts M1

$\int_1^2 kx^3\,dx \left[\frac{1}{4}kx^4\right]_1^2 = 4k - \frac{1}{4}k$ oe both answer correct A1

$\frac{1}{4} + 4k - \frac{1}{4}k = 1$ adding two answers and putting = 1 dM1dep on previous M

$\frac{15k}{4} = \frac{3}{4}$

$k = \frac{1}{5}$ * A1 4

M1 for adding two integrals together = 1, ignore limits

A1 for correct integration, ignore limits

M1 using correct limits

A1 cso

(b)	$\int_0^1 \frac{1}{2}x^2\mathrm{d}x = \left[\frac{1}{6}x^3\right]_0^1 = \frac{1}{6}$	attempt to integrate xf(x) for one part	M1	
		1/6	A1	
	$\int_1^2 \frac{1}{5}x^4\mathrm{d}x = \left[\frac{1}{25}x^5\right]_1^2 = \frac{32}{25} - \frac{1}{25}$			
	$= \frac{31}{25}$ or 1.24		A1	
	$E(X) = \frac{1}{6} + \frac{31}{25}$			
	$= \frac{211}{150} = 1\frac{61}{150} = 1.40\dot{6}$		A1	4

M1 attempting to use integral of x f(x)
A1 correct two integrals added with limits
A1 correct integration ignore limits
A1 awrt 1.41

(c)	F(x) = $\int_0^x \frac{1}{2}t\mathrm{d}t$ (for $0 \le x \le 1$)	ignore limits for M	M1	
	$= \frac{1}{4}x^2$	must use limit of 0	A1	
	F(x) = $\int_1^x \frac{1}{5}t^3\mathrm{d}t; + \int_0^1 \frac{1}{2}t\mathrm{d}t$ (for $1 < x \le 2$)	need limit of 1 and variable upper limit; need limit 0 and 1	M1; M1	
	$= \frac{1}{20}x^4 + \frac{1}{5}$		A1	
	$\mathrm{F}(x)\begin{cases} 0 & x < 0 \\ \frac{1}{4}x^2 & 0 \le x \le 1 \\ \frac{1}{20}x^4 + \frac{1}{5} & 1 < x \le 2 \\ 1 & x > 2 \end{cases}$			
		middle pair	B1ft	
		ends	B1	7

M1 Att to integrate $\frac{1}{2}$t (they need to increase the power by 1).
Ignore limits for method mark
A1 $\frac{1}{4}x^2$ allow use of t. must have used/implied use of limit of 0.
This must be on its own without anything else added
M1 att to integrate $\int_1^x \frac{1}{5}t^3 \mathrm{d}t$ and correct limits.
M1 $\int_0^1 \frac{1}{2}t\mathrm{d}t +$ Att to integrate using limits 0 and 1.
no need to see them put 0 in.
they must add this to their $\int_1^x \frac{1}{5}t^3 \mathrm{d}t$. may be given if they add ¼

$$\left(\begin{array}{l}\text{Alternative method for these last two M marks} \\ \text{M1 for att to } \int \frac{1}{5}t^3 \mathrm{d}t \text{ and putting } + \mathrm{C} \\ \text{M1 use of } \mathrm{F}(2) = 1 \text{ to find C}\end{array}\right)$$

A1 $\frac{1}{20}x^4 + \frac{1}{5}$ must be correct
B1 middle pair followed through from their answers.
condone them using < or ≤ incorrectly they do not need to match up
B1 end pairs. condone them using < or ≤. They do not need to match up

NB if they show no working and just write down the distribution. If it is correct they get full marks. If it is incorrect then they cannot get marks for any incorrect part. So if $0 < x < 1$ is correct they can get M1 A1 otherwise M0 A0. if $3 < x < 4$ is correct they can get M1 A1A1 otherwise M0 A0A0. you cannot award B1ft if they show no working unless the middle parts are correct.

(d)	F(m) = 0.5	either eq	M1	
	$\frac{1}{20}m^4 + \frac{1}{5} = 0.5$	eq for their $1 \le x \le 2$	A1ft	
	$m = \sqrt[4]{6}$ or 1.57 or awrt 1.57		A1	3

M1 either of their $\frac{1}{4}x^2$ or $\frac{1}{20}x^4 + \frac{1}{5} = 0.5$
A1 for their F(X) $1 < x < 2 = 0.5$
A1 cao

If they add both their parts together and put = 0.5 they get M0
If they work out both parts separately and do not make the answer clear they can get M1 A1 A0

	(e)	negative skew	B1	
		This depends on the previous B1 being awarded. One of the following statements which must be compatible with negative skew and their figures. If they use mode then they must have found a value for it Mean < Median Mean < mode Mean < median (< mode) Median < mode Sketch of the pdf	dB1	2
		B1 negative skew only B1 Dependent on getting the previous B1. their reason must follow through from their figures.		

[20]

9.	(a)	$K(2^4 + 2^2 - 2) = 1$	M1	
		$K = 1/18$	A1	2
		M1 putting F(2) = 1 or F(2) – F(1) = 1		
		A1 cso. Must show substituting y = 2 and the 1/18		
	(b)	$1 - F(1.5) = 1 - \frac{1}{18}(1.5^4 + 1.5^2 - 2)$	M1	
		$= 0.705$ or $\frac{203}{288}$	A1	2
		M1 either attempting to find 1 – F(1.5) may write and use F(2) – F (1.5)		
		A1 awrt 0.705		

(c) $f(y) = \begin{cases} \frac{1}{9}(2y^3 + y) & 1 \le y \le 2 \\ 0 & otherwise \end{cases}$ M1A1

B1 3

M1 attempting to differentiate. Must see either a $y^n \to y^{n\text{-}1}$ at least once

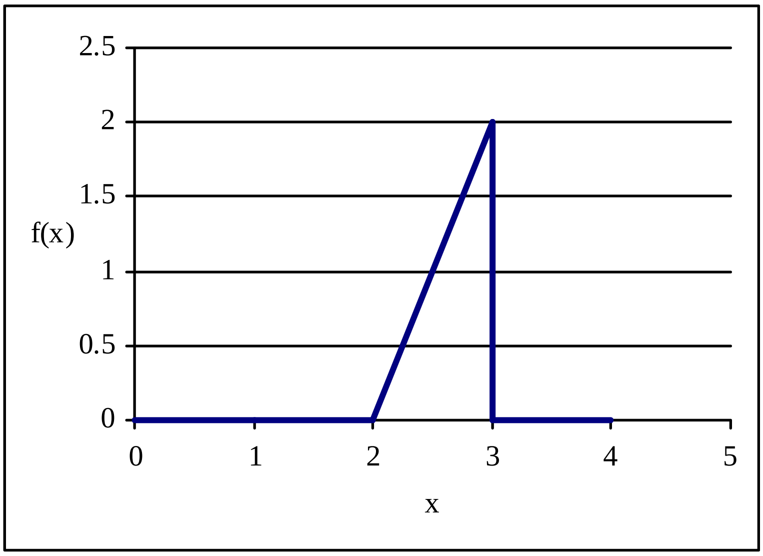
A1 for getting $\frac{1}{9}(2y^3 + y)$ o.e and $1 \le y \le 2$ allow $1 < y < 2$

B1 for the 0 *otherwise.* Allow 0 for $y < 1$ and 0 for $y > 2$

Allow them to use any letter

[7]

10. (a)



Max height of 2 labelled and goes through (2, 0) B1

shape must be between 2 and 3 and no other lines drawn (accept patios drawn) B1

/ correct shape B1 3

B1 the graph must have a maximum of 2 which must be labelled

B1 the line must be between 2 and 3 with not other line drawn except patios. They can get this mark even if the patio cannot be seen.

B1 the line must be straight and the right shape.

(b) 3 B1 1

B1 Only accept 3

(c)	$\int_2^3 2x(x-2)\mathrm{d}x = \left[\frac{2x^3}{3} - 2x^2\right]_2^3$	M1A1	
	$= 2\frac{2}{3}$	A1	3

M1 attempt to find $\int x\mathrm{f}(x)\mathrm{d}x$ for attempt we need to see $x^n \to x^{n+1}$. ignore limits

Al correct integration ignore limits

Al accept $2\frac{2}{3}$ or awrt 2.67 or $2.\dot{6}$

(d)	$\int_2^m 2(x-2)\mathrm{d}x = 0.5$	M1	
	$\left[x^2 - 4x\right]_2^m = 0.5$		
	$m^2 - 4m + 4 = 0.5$	A1	
	$m^2 - 4m + 3.5 = 0$		
	$m = \frac{4 \pm \sqrt{2}}{2}$	M1	
	$m = 2.71$	A1	4

M1 using $\int \mathrm{f}(x)\mathrm{d}x = 0.5$

A1 $m^2 - 4m + 4 = 0.5$ oe

M1 attempting to solve quadratic.

Al awrt 2.71 or $\frac{4+\sqrt{2}}{2}$ or $2+\frac{\sqrt{2}}{2}$ oe

(e)	Negative skew.	B1	
	mean < median < mode	B1dep	2

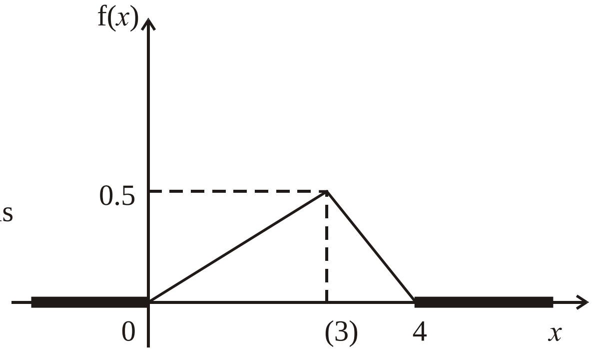
First B1 for negative

Second B1 for mean < median< mode.
Need all 3 or may explain using diagram.

[13]

11. (a)

axis



	(0)*, 4, 0.5	*0 may be implied by start at y	B1	
	both patio		B1	
	must be straight		B1	3
(b)	Mode is $x = 3$		B1	1
(c)	$F(x) = \int_0^x \frac{1}{6}t\,dt$ (for $0 \le x \le 3$)	ignore limits for M	M1	
	$= \frac{1}{12}x^2$	must use limit of 0	A1	
	$F(x) = \int_3^x 2 - \frac{1}{2}t\,dt; + \int_0^5 \frac{1}{6}t\,dt$ (for $3 < x \le 4$)			
		need limit of 3 and variable upper limit; need limit 0 and 3	M1; M1	
	$= 2x - \frac{1}{4}x^2 - 3$		A1	
	$F(x)\left\{\begin{array}{cc} 0 & x < 0 \\ \frac{1}{12}x^2 & 0 \le x \le 3 \\ 2x - \frac{1}{4}x^2 - 3 & 3 < x \le 4 \\ 1 & x > 4 \end{array}\right\}$	middle pair	B1ft	
		ends	B1	7
(d)	$f(m) = 0.5$	either eq	M1	
	$\frac{1}{12}x^2 = 0.5$	eq for their $0 \le x \le 3$	A1ft	
	$x = \sqrt{6} = 2.45$	$\sqrt{6}$ or awrt 2.45	A1	3

[14]

12. (a) $f(x) = \begin{cases} \dfrac{1}{\beta - \alpha}, & \alpha < x < \beta, \\ 0, & \text{otherwise} \end{cases}$ function including inequality, 0 otherwise B1,B1 2

(b) $\dfrac{\alpha + \beta}{2} = 2, \dfrac{3 - \alpha}{\beta - \alpha} = \dfrac{5}{8}$ or equivalent B1, B1

$\alpha + \beta = 4$
$3\alpha + 5\beta = 24$

$3(4 - \beta) + 5\beta = 24$
$2\beta = 12$ attempt to solve 2 eqns M1
$\beta = 6$

$\alpha = -2$ both A1 4

(c) $E(X) = \dfrac{150 + 0}{2} = 75$ cm 75 B1 1

(d) Standard deviation = $\sqrt{\dfrac{1}{12}(150 - 0)^2}$ M1

= 43.30127... cm $25\sqrt{3}$ or awrt 43.3 A1 2

(e) $P(X < 30) + P(X > 120) = \dfrac{30}{150} + \dfrac{30}{150}$

1st or at least one fraction, + or double M1, M1

$= \dfrac{60}{150}$ or $\dfrac{2}{5}$ or 0.4 or equivalent fraction A1 3

[12]

13. (a) $1 - F(0.3) = 1 - (2 \times 0.3^2 - 0.3^3)$ 'one minus' required M1
= 0.847 A1 2

(b) $F(0.60) = 0.5040$
$F(0.59) = 0.4908$ both required awrt 0.5, 0.49 M1A1

0.5 lies between therefore median value lies between 0.59 and 0.60. B1 3

(c) $f(x) = \begin{cases} -3x^2 + 4x, & 0 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$ attempt to differentiate, all correct M1A1 2

	(d)	$\int_0^1 x\mathrm{f}(x)\mathrm{d}x = \int_0^1 -3x^3 + 4x^2\mathrm{d}x$	attempt to integrate $x\mathrm{f}(x)$	M1	
		$= \left[\dfrac{-3x^2}{4} + \dfrac{4x^3}{3}\right]_0^1$	sub in limits	M1	
		$= \dfrac{7}{12}$ or $0.58\dot{3}$ or 0.583 or equivalent fraction		A1	3
	(e)	$\dfrac{\mathrm{df}(x)}{\mathrm{d}x} = -6x + 4 = 0$ attempt to differentiate f(x) and equate to 0		M1	
		$x = \dfrac{2}{3}$ or $0.\dot{6}$ or 0.667		A1	2
	(f)	mean < median < mode, therefore negative skew.	Any pair, cao	B1,B1	2

[14]

14.	(a)	$\int_1^4 \dfrac{1+x}{k}\,dx = 1\,k$	$\int \mathrm{f}(x) = 1$	M1	
		Area = 1			
		$\therefore \left[\dfrac{x}{k} + \dfrac{x^2}{2k}\right]_1^4 = 1$		A1	
		correct integral / correct expression			
		$k = \dfrac{21}{2}$	cso	A1	3
	(b)	$\mathrm{P}(X \le x_0) = \int_1^{x_0} \dfrac{2}{21}(1+x)$		M1	
		$\int \mathrm{f}(x)$			
		variable limit or +C			
		$\left[\dfrac{2x}{21} + \dfrac{x^2}{21}\right]_1^{x_0}$		A1	
		correct integral + limit of 1			
		$= \dfrac{2x_0 + x_0^2 - 3}{21}$ or $\dfrac{(3+x)(x-1)}{21}$		A1	
		May have k in			

$$\mathrm{F}(x) = \begin{cases} 0, & x<1 \\ \dfrac{x^2+2x-3}{21} & 1\le x<4 \\ 1 & x\ge 4 \end{cases}$$ B1ft; B1 5

middle; ends

(c) $\mathrm{E}(X)=\int_1^4 \frac{2x}{21}(1+x)\mathrm{d}x$ M1

valid attempt $\int x\mathrm{f}(x)$

x^2 and x^3

$=\left[\frac{x^2}{21}+\frac{2x^3}{63}\right]_1^4$ A1

correct integration

$=\frac{171}{63}=2\frac{5}{7}=\frac{19}{7}=2.7142....$ A1 3

awrt 2.71

(d) $\mathrm{F}(m)=0.5 \Rightarrow \frac{x^2+2x-3}{21}=\frac{1}{2}$ M1

putting their F(x) = 0.5

$2x^2 + 4x – 27 = 0$ or equiv

$\therefore x=\frac{-4\pm\sqrt{16-4.2(-27)}}{4}$ M1

attempt their 3 term

quadratic

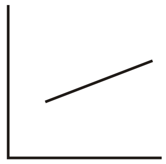
$\therefore x=-1\pm 3.80878....$

i.e. $x = 2.8078....$ A1 3

awrt 2.81

(e) Mode = 4 B1 1

	(f)	<u>Mean < median < mode</u> ($\Rightarrow$ negative skew) Or <u>Mean < median</u> *allow numbers in place of words*		B1	1



w diagram but line must not cross y axis

[16]

15.	(a)	$\int_2^3 kx(x-2)\,dx = 1$	$\int f(x) = 1$	M1	
		$\left[\frac{1}{3}kx^3 - kx^2\right]_2^3 = 1$	attempt $\int$ need either x^3 or x^2	M1	
			correct $\int$	A1	
		$(9k - 9k) - (\frac{8k}{3} - 4k) = 1$			
		$k = \frac{3}{4} = 0.75$	cso	A1	4
	(b)	$\mathrm{E}(X) = \int_2^3 \frac{3}{4}x^2(x-2)\mathrm{d}x$	attempt $\int xf(x)$	M1	
		$\left[\frac{3}{16}x^4 - \frac{1}{2}x^3\right]_2^3$	correct $\int$	A1	
		$= 2.6875 = 2\frac{11}{16} = 2.69$ (3sf)	awrt 2.69	A1	3

(c)	$F(x) = \int_2^x \frac{3}{4}(t^2 - 2t)dt$	$\int f(x)$ with variable limit or +C	M1	
	$\left[\frac{3}{4}\left(\frac{1}{3}t^3 - t^2\right)\right]_2^x$	correct integral lower limit of 2 or F(2) = 0 or F(3)	A1	
		= 1	A1	
	$= \frac{1}{4}(x^3 - 3x^2 + 4)$		A1	
	$F(x) = \begin{cases} 0 & x \le 2 \\ \frac{1}{4}(x^3 - 3x^2 + 4) & 2 < x < 3 \\ 1 & x \ge 3 \end{cases}$	middle, ends	B1ft, B1	6
	$F(x) = \frac{1}{2}$			
(d)	$\frac{1}{4}(x^3 - 3x^2 + 4) = \frac{1}{2}$	their F(x) = 1/2	M1	
	$x^3 - 3x^2 + 2 = 0$			
	$x = 2.75$, $x^3 - 3x^2 + 2 > 0$			
	$x = 2.70$, $x^3 - 3x^2 + 2 < 0$root between 2.70 and 2.75		M1	2
	(or F(2.7) = 0.453, F(2.75) = 0.527 median between 2.70 and 2.75			

[15]

16.	(a)	$\int_0^2 k(4x - x^3)\,dx = 1$		M1 A1	
			$\int f(x)dx$ = 1, all correct		
		$k\left[2x^2 - \frac{1}{4}x^4\right]_0^2 = 1$ (*)		A1	
		$k(8 - 4) = 1$		cso A1	4
		$k = \frac{1}{4}$			

(b) $E(X) = \int_0^2 x.\frac{1}{4}(4x - x^3)dx$ M1

$\int xf(x)dx$

$= \left[\frac{1}{3}x^3 - \frac{1}{20}x^5\right]_0^2$ (*)

$= \frac{16}{15}$ A1 3

1.07 or $1\frac{1}{15}$ or $\frac{16}{15}$ or $1.0\dot{6}$

(c) At mode, $f'(x) = 0$ M1

Implied

$4 - 3x^2 = 0$ M1

Attempt to differentiate

$x = \frac{2}{\sqrt{3}}$ A1 3

$\sqrt{\frac{4}{3}}$ or 1.15 or $\frac{2}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3}$

(d) At median, $\int_0^x \frac{1}{4}\left(4t - t^3\right)dt = \frac{1}{2}$ M1

$F(x) = \frac{1}{2}$ or $\int f(x)dx = \frac{1}{2}$

$\frac{1}{4}\left(2x^2 - \frac{1}{4}x^4\right) = \frac{1}{2}$ M1

Attempt to integrate

$x^4 - 8x^2 + 8 = 0$

$x^2 = 4 \pm 2\sqrt{2}$ M1

Attempt to solve quadratic

$x = 1.08$ A1 4

Awrt 1.08

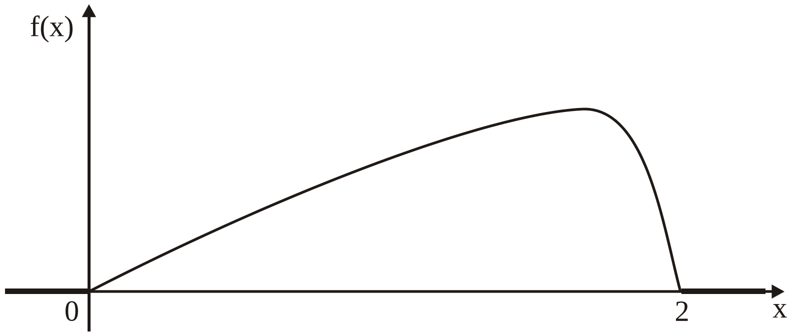
[14]

(e) mean (1.07) < median (1.08) < mode (1.15) M1

any pair

$\Rightarrow$ negative skew cao A1 2

(f)



lines x<0 and x >2, labels, 0 and 2 B1

negative skew between 0 and 2 B1 2

[18]

17. (a) $k\int_1^4 (-x^2 + 5x - 4)\,dx = 1$ M1

use of $\int f(x)dx = 1$

$\therefore k\left[-\frac{x^3}{3} + \frac{5x^2}{2} - 4x\right]_1^4 = 1$ A1

All correct integ. with limits

(*)$\Rightarrow k = \frac{2}{9}$ (*) A1 3

c.s.o.

(b) $E(X) = \int_1^4 \tfrac{2}{9}(-x^3 + 5x^2 - 4x)\mathrm{d}x$ — M1

use of $\int xf(x)\mathrm{d}x$

$= \frac{2}{9}\left[-\frac{x^4}{4} + \frac{5x^3}{3} - \frac{4x^2}{2}\right]_1^4$ — A1

Correct integ^n with limits

$= \underline{\frac{5}{2}}$ — A1 — 3

cao

(c) $\frac{\mathrm{d}}{\mathrm{d}x} f(x) = \frac{2}{9}(-2x + 5) = 0; \Rightarrow \text{Mode} = \frac{5}{2}$ — M1, A1 — 2

Diff^n of f(x)
& = 0

(d) $F(x) = \int_1^{x_0} \frac{2}{9}(-x^2 + 5x - 4)\mathrm{d}x$ — M1

use of $\int f(x)\mathrm{d}x$

$= \left[\frac{2}{9}\left(-\frac{x^3}{3} + \frac{5x^2}{2} - 4x\right)\right]_1^{x_0}$ — A1

Integ^n with limits
1 & symbol

$= \frac{2}{9}\left\{-\frac{x_0^3}{3} + \frac{5x_0^2}{2} - 4x_0 + \frac{11}{6}\right\}$ — A1

$\therefore F(x) = \begin{cases} 0 \\ \tfrac{2}{9}\left(-\frac{x^3}{3} + \frac{5x^2}{2} - 4x + \frac{11}{6}\right) \\ 1 \end{cases}$ — B1 B1 — 5

x < 1 *x < 1; x > 4*
1 ≤ x ≤ 4 *1 ≤ x ≤ 4*
x > 4

(e) $P(X \leq 2.5) = F(2.5) = 0.5$ — M1 A1 — 2

F(2.5) or integral etc

(f) Median = 2.5; Distribution is symmetrical — B1cao, B1cao — 2

[17]

18 (a) $E(X) = \int_0^1 \frac{1}{3}x\,dx + \int_1^2 \frac{8x^4}{45}\,dx$ M1M1

∫xf(x)dx, 2 terms added

$= \left[\frac{1}{6}x^2\right]_0^1 + \left[\frac{8x^5}{225}\right]_1^2$ A1A1

Expressions, limits

$= 1.26\dot{8} = 1.27$ to 3 sf or $\frac{571}{450}$ or $1\frac{121}{450}$ A1 5

awrt 1.27

(b) $F(x_0) = \int_0^{x_0} \frac{1}{3}\,dx = \frac{1}{3}x_0$ for $0 \le x < 1$ M1A1

variable upper limit on ∫f(x)dx, $\frac{1}{3}x_0$

$F(x_0) = \frac{1}{3} + \int_1^{x_0} \frac{8x^3}{45}\,dx$ for $1 \le x \le 2$ M1

their fraction + v.u.l on ∫f(x)dx &2 terms

$= \frac{1}{3} + \left[\frac{8x^4}{180}\right]_1^{x_0}$ A1

$= \frac{1}{45}(2x_0^4 + 13)$ A1

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{3}x & 0 \le x < 1 \\ \frac{1}{45}(2x^4 + 13) & 1 \le x \le 2 \\ 1 & x > 2 \end{cases}$$ B1,B1 7

middle pair, ends

(c) $F(m) = 0.5$

$\frac{1}{45}(2x^4 + 13) = \frac{1}{2}$ M1A1ft

Their function = 0.5

$m^4 = 4.75$

$m = 1.48$ to 3sf A1 3

awrt 1.48

(d) mean < median B1

Negative Skew dep B1 2

[17]

19. (a) $\int_0^4 kx(5-x)\,dx = 1$ M1

Limits required

$k\left[\frac{5x^2}{2} - \frac{x^3}{3}\right]_0^4 = 1$ A1

$\left[\frac{5x^2}{2} - \frac{x^3}{3}\right]$

Sub in limits and solve to give ****$k = \frac{3}{56}$ **** A1 3

Correct solution

(b) $F(x) = \int_0^{x_0} f(x)dx = \int_0^{x_0} \frac{3}{56}x(5-x)dx = \frac{3}{56}\left[\frac{5x^2}{2} - \frac{x^3}{3}\right]_0^{x_0}$ M1

Variable upper limit required

$= \frac{x_0^2}{112}(15 - 2x_0)$ A1

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{112}(15-2x) & 0 \le x \le 4 \\ 1 & x > 4 \end{cases}$$

Ends, middle. B1,B1ft 4

(c) $E(x) = \int_0^4 \frac{3}{56}x^2(5-x)dx = \frac{3}{56}\left[\frac{5x^3}{3} - \frac{x^4}{4}\right]_0^4 = 2.29 \int xf(x)dx, \left[\frac{5x^3}{3} - \frac{x^4}{4}\right],$

3 sf $(2\frac{2}{7})$ M1A1A1 3

(d)	$f'(x) = \frac{3}{56}(5 - 2x) = 0 \Rightarrow$ Mode = 2.5	M1A1A1	3	
	Attempt f′(x), (5 – 2x) = 0, 2.5 *(Or Sketch M1, x = 0 & 5 A1, Mode = 2.5 A1)*			
(e)	F(2.3) = 0.491, F(2.5) = 0.558	M1,A1		
	Their F, awrt 0.491 & 0.558 or 0.984 & –6.5			
	$F(m) = 0.5 \Rightarrow m$ lies between 2.3 and 2.5	cso A1	3	
(f)	Mean (2.29) < Median (2.3 – 2.5) < Mode (2.5)	B1		
	Negative skew	B1 dep	2	
				[18]

20.

(a)	$\int_{-1}^{0} k(x^2 + 2x + 1)\,dx = 1$ (limits needed and = 1)	M1		
	$\left[k\left(\frac{x^3}{3} + x^2 + x\right)\right]_{-1}^{0} = 1$ attempt at integration	M1 A1		
	$\underline{k = 3}$ (*)	A1	4	
(b)	$E(X) = \int_{-1}^{0} x.f(x)\,dx$	M1		
	$= \int_{-1}^{0} (3x^3 + 6x^2 + 3x)\,dx$ limits needed	A1		
	$= \left[\frac{3x^4}{4} + 2x^3 + \frac{3x^2}{2}\right]_{-1}^{0}$ integration and substituting limits	dep M1		
	$= \underline{-\frac{1}{4}}$	A1	4	
(c)	$\int_{-1}^{x_0} (3x^3 + 6x^2 + 3x)\,dx = \left[x^3 + 3x^2 + 3x\right]_{-1}^{x_0}$	M1		
	$= x_0 + 3x_0^2 + 3x_0 + 1$	A1		
	$F(x) = \begin{cases} 0 & x < -1 \\ x^3 + 3x^2 + 3x + 1 & -1 \le x \le 0 \\ 1 & x > 0 \end{cases}$	B1 B1	4	
(d)	$P(-0.3 < X < 0.3) = F(0.3) - F(-0.3)$	M1		
	= 1 – 0.343	A1		
	= $\underline{0.657}$	A1	3	
				[15]

21. (a) $P(X > 0.7) = 1 - F(0.7) = 0.4267$ M1 A1 2

(b) $f(x) = \dfrac{d}{dx}F(x) = \dfrac{4}{3} \times 2x - \dfrac{4x^2}{3}$ M1

$= \dfrac{4x}{3}(2 - x^2)$ for $0 \le x \le 1$ A1 2

(c) $E(X) = \displaystyle\int_0^1 \frac{4}{3}(2x^2 - x^4)\,dx = \left[\frac{4}{3}\left(\frac{2x^3}{3} - \frac{x^5}{5}\right)\right]_0^1$ M1 A1

$= \dfrac{28}{45} = 0.622$ A1

$\text{Var}(X) = \displaystyle\int_0^1 \frac{4}{3}(2x^3 - x^5)\,dx - \left(\frac{28}{45}\right)^2$ M1

$= \left[\dfrac{4}{3}\left(\dfrac{2x^4}{4} - \dfrac{x^6}{6}\right)\right]_0^1 - \left(\dfrac{28}{45}\right)^2$ A1

$= \dfrac{116}{2025} = 0.05728$ A1 6

(d) $f(x) = \dfrac{4}{3}(2 - 3x^2) = 0$ M1

$\Rightarrow$ mode $= \sqrt{\dfrac{2}{3}} = 0.816496$ A1

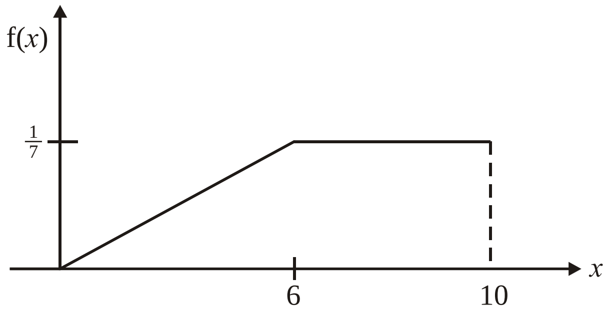
skewness $= \dfrac{\dfrac{28}{45} - \sqrt{\dfrac{2}{3}}}{\sqrt{\dfrac{116}{2025}}} = -0.81170$ M1 A1 4

[14]

$\frac{x}{42}$ B1

$\frac{1}{7}$ B1

22. (a)



Labelled axes and < 0, > 10	B1		
$\frac{1}{7}$, 6, 10	B1	4	

(b) $P(X \geq 5) = 1 - P(X < 5)$

5 and ∫ or area Δ	M1		
$= 1 - \frac{5}{42} \times \frac{1}{2} \times 5$ (area of Δ)			
full method	M1		
$= 1 - \frac{25}{84}$ $\qquad = \frac{59}{84}$	A1	3	
Probability it does *not* break down is $\left(\frac{59}{84}\right)^2$	M1		
$\therefore$ probability it does break down is $1 - \left(\frac{59}{84}\right)^2$ = (awrt) 0.507	A1	2	

[9]

23. (a)

$F(1.5) = 1 \Rightarrow k(2 \times (1.5)^3 - (1.5)^4) = 1$	M1	
i.e. $k\left[2 \times \frac{27}{8} - \frac{81}{16}\right] = 1$		
i.e. $k\left(\frac{108-81}{16}\right) = 1$ $\qquad \therefore k = \frac{16}{27}$ (*)	A1 cso	2

(b)

$P(T > 1) = 1 - F(1)$, $\quad = 1 - \frac{16}{27}(2 - 1) = \frac{11}{27}$	M1, A1	2

(c)	$f(t) = F'(t) = , \frac{16}{27}(6t^2 - 4t^3)$		M1, A1	
	i.e. $f(t) = \begin{cases} \frac{32}{27}(3t^2 - 2t^3) & 0 \le t \le 1.5 \\ 0 & \text{otherwise} \end{cases}$			
	Full definition		B13	
(d)	$E(T) = \int_0^{1.5} t\,f(t)\,dt \;=\; \frac{32}{27}\int_0^{1.5}(3t^3 - 2t^4)\,dt$	$\int t\,f(t)$	M1	
	$= \frac{32}{27}\left[\frac{3t^4}{4} - \frac{2t^5}{5}\right]_0^{\frac{3}{2}}$		A1	
	$= \frac{32}{27}\left[\left(\frac{243}{64} - \frac{2}{5} \times \frac{243}{32}\right) - (0)\right]$			
	$= \frac{9}{2} - \frac{18}{5} \quad = \quad 0.9 \qquad (*)$		A1 cso	3
(e)	$F(E(T)) = \frac{16}{27}(2 \times 0.9^3 - 0.9^4) = 0.4752$			
	evidence seen		B1	
(f)	$P(T > 1 \mid T > 0.9) \;=\; \frac{P(T > 1)}{P(T > 0.9)}, \;=\; \frac{\text{part }(b)}{1 - \text{part }(e)}, = 0.7763\ldots$		M1, M1	
	accept awrt 0.776		A13	

[14]

1. Students seemed to fare better on the continuous distributions with the material spread over two questions rather than all together as previously.

In part (a) a large majority of candidates were able to set F(m) = 0.5 and formulate a correct equation. However then many candidates were unable to manipulate the initial equation into a more suitable form. However candidates who showed their working were able to earn credit for using a correct method on an incorrect equation. Candidates who solved the quadratic on their calculator showed no such method and therefore lost two marks instead of one for the incorrect answer. A variety of methods were then used successfully to solve the equation namely ‘the formula’, ‘completing the square’ and ‘trial and improvement’.

Part (b) was well done with only a few candidates neglecting to put “0 otherwise” in the full definition.

In part(c) the main problem was the result of confusion between discrete and continuous variables. It was not uncommon to see $P(X \geq 2) = 1 - F(1.1)$

Many candidates were able to gain the method mark in part (d). Those who didn’t put $(0.6267)^4$ either repeated their answer to part (c) or multiplied it by 4, seemingly unconcerned by a probability greater than 1.

2. More candidates seemed to score full marks or nearly full marks than usual for this type of question. Some had problems with the concept of proof and some circular arguments were seen in part (b). There were also some problems in manipulating the algebraic fractions

In part (a) many candidates used a proof by contradiction approach rather than starting from f(y) > 0. Some wrongly thought that a probability density function cannot be 0 at any point and some thought that it can’t be greater than 1. More attempted an explanation in words than a symbolic proof.

Part (b) saw many excellent solutions. There was a lot of detail involved. Yes, the equation to be solved was only linear, but the coefficients were potentially forbidding to those of us who only use a calculator as a last resort. There were many admirable responses, where candidates displayed persistence and excellent command of detail. The quantity of algebraic working seen varied substantially, from a few lines of genuine, succinct and accurate work, to a few pages of laboured inaccurate solutions.

Part (c) was surprisingly badly done. A few candidates were confused by the variable being y rather than the more usual x and so reflected their sketch in $y = x$. The minority who took their time over the sketch got it correct while those who just saw the squared term assumed a parabola intersecting the x-axis at 0 and 3. The mode was usually identified from their sketch although, as ever, there were those who gave the y value rather than the x value.

3. Most of candidates were able to gain the majority of the marks for this question. In part (a) most candidates either correctly substituted 0 into the formula for F(x) or used F(0) – F(–2).
A common error was to integrate F(X), which in many cases resulted in a correct answer but gained no marks as the method was incorrect. There were also a number of students who believed the distribution to be discrete and calculated F(x) accordingly.

In part (b) there were a few candidates who integrated F(x) or used F(x) rather than differentiating to find f(x). Some of those who differentiated correctly then failed to identify the regions in which the values of $\frac{1}{6}$ and 0 were valid.

In part (c) a large number of candidates were unable to completely state the name of the distribution with common errors being to omit either the word ‘continuous’ or the word ‘uniform’. In part (d) although most candidates were able to identify or calculate the mean, a few carried out complicated unnecessary calculations, which were usually incorrect. The candidates that used the formula usually achieved a correct solution for the variance. However, those that attempted to use $\int x^2 \mathrm{f}(x)$ – mean^2 often made errors or forgot to subtract the mean squared.

In part (d) many candidates failed to realise that the probability that X equals a single value in a continuous distribution is always 0.

4. Many candidates scored full marks in part (a). However, there was inevitably substantial variation in the style and presentation of their arguments. A small number of scripts were models of clarity and economy, while other candidates were not just long-winded and confusing, but their scripts contained incorrect and contradictory statements (e.g. $6k = 1$, $3k = 1$, $9k = 1$) on the way to a correct final solution.
It is reassuring that many perfect solutions to part (b) were seen. The most common problem was the part of the distribution dealing with3 ≤ x ≤ 4 where many candidates simply worked out $\int_3^x \frac{1}{3}\mathrm{d}t = \frac{x}{3} - 1$. Of those who got the correct answer a mixture of methods were used.

Some use the approach $\int_3^x \frac{1}{3}\mathrm{d}t + \mathrm{F}(3)$ where $\mathrm{F}(3) = \frac{2}{3}$, while others chose indefinite integration: $\int \frac{1}{3}\mathrm{d}x = \frac{x}{3} + \mathrm{C}$ and $\mathrm{F}(4) = 1$.

In part (c) correct answers seemed relatively elusive. A significant number of candidates attempted to find E(X) by using $\int x\mathrm{f}(x)\mathrm{d}x$ for one part only, usually the f(x) for $3 \le x \le 4$.

There were a small number of candidates who 'averaged' their answers to the two parts: $\dfrac{5/4 + 7/6}{2}$. Other candidates multiplied the F(x) by x before integrating.

Part (d) was well done by many candidates, even when there had been problems earlier in the question. Most understood what they were trying to do, and often had a correct version of the function to hand. However, some failed to provide an acceptable conclusion: "so the median is between the two numbers" or "F(2.6) < m < F(2.7)" are examples which did not gain the mark. The most common error was to amalgamate their two functions from part (b). Others used the 'wrong' function, i.e. their function from (b) for$3 \le x \le 4$.
Those candidates who used calculators to solve the cubic equation usually provided the required amount of supporting detail.

5. A minority of candidates achieved a high rate of success on this question.

In part (a) most candidates were able to write E(X) = 2 without difficulty.

A variety of methods were seen in part (b). The method of the mark scheme was seen, perhaps only from a minority of candidates. Many candidates preferred to use calculus: $\int \mathrm{f}(x)\mathrm{d}x = 1$.

However, the use of calculus requires more subtlety and sensitivity than was available to many of the candidates. Answers of a = ½ and b = 2 seemed to be not uncommon, resulting from the incorrect methods: $\int_0^2 ax\,\mathrm{d}x = 1$ and $\int_2^4 (b - ax)\mathrm{d}x = 1$.

There were candidates who obtained the correct answers using calculus, but it often took considerable working, in contrast to the expected method.

There were some candidates who obtained full marks to part (c) with solutions that were confident, fluent and accurate. Furthermore, many of these responses were also efficient: four or five lines of working provided a solution that was not just correct but contained all the required details. However, a wide variety of alternative responses were also seen. Some were indeed correct, but inefficient. Other candidates used an incorrect strategy. Some candidates only worked with the domain $2 \le x \le 4$. Others worked with both domains, but wanted to keep the domains separate, resulting in two separate versions of $\mathrm{Var}(X)$

: $\mathrm{Var}(X) = \int_0^2 x^2 \frac{1}{4}x\,\mathrm{d}x - \left(\int_0^2 x\frac{1}{4}x\,\mathrm{d}x\right)^2$ and $\mathrm{Var}(X) = \int_0^4 x^2\left(x - \frac{1}{4}x\right)\mathrm{d}x - \left(\int_0^4 x\left(x - \frac{1}{4}x\right)\mathrm{d}x\right)^2$

Some candidates then calculated the average of these two versions of the variance.

Many candidates also found E(X) from scratch in this part rather than using the answer they had in part (a). Not only did this waste time, but whilst they often had it correct in part (a) they gained an incorrect value by integration in this part which they then went on to use.

It must be noted that where the answer to a question is given, marks cannot be gained by restating this without sufficient working. Some attempts were made to describe the answer as proven even though no real working had been done.

A reasonable number of correct solutions to part (d) were seen. Some candidates went so far as to specify fully the cumulative distribution function before using the correct part to find the lower quartile. Even though this extra work was not required, strictly speaking, it did provide these candidates with a ready method for part (e).

It would appear that whilst most candidates attempted part (e), their responses consisted of a simple statement, usually “greater than 0.5” together with an irrelevant reason. A tiny minority of candidates responded in the manner intended. A few provided a clear diagram to illustrate this same argument. However, the majority of successful candidates preferred to evaluate the probability. This was not straightforward, except for those who had already obtained a full and correct version of the cumulative distribution function. Part (e) seemed to challenge all but the most able candidates.

6. Part (a), with its ‘answer given’, produced fewer problems than similar questions in previous papers. Most candidates were able to obtain the required value of k.

Part (b) was generally well done. There were a wide variety of methods used such as finding the area of a trapezium, others found the area of the triangle and subtracted from 1. Others obtained F(x). The most common fault was the use of incorrect limits, 7 was often seen as the lower limit.

Part (c) was a good source of marks for a majority of candidates. A few lost marks as a result of not writing their answers as an exact number. However, many provided answers as both exact fractions and as approximated decimals. The most common error was to findE(T^2), call it Var(T) and then stop.

Part (d) was not popular. Of those who attempted it, there were some long-winded methods involving calculus and ultimately incorrect answers. The most successful candidates did a quick sketch of the p.d.f. to find the mode.

There were a few good sketches in part (e) however; there were all sorts of alternatives. Many just gave a sketch of the original p.d.f.

7. This question proved challenging in parts to some candidates but was attempted in full by many, with a high degree of success.

In part (a) most candidates were aware that they needed to integrate the given function and did so successfully, including the fractions. Problems generally arose in the use of the correct limits. It was common to see candidates use limits of 0 or 1 and 4 rather than using a variable upper limit. Several candidates chose to use a constant c rather than limits but often did not proceed to use F(4) = 1 or F(1) = 0 to find the value of c. A large number of candidates who got the correct answer went on to multiply their expression by 9.

In (b) F(x) was defined well – candidates seem to be more aware of the need for the 0 and 1 and there were a limited number who had the wrong ranges for these.

The majority of correct answers in (c) were found by solving the quadratic rather than by the easier method of substituting 2.5 into the equation. Many of those who used the quadratic formula used complicated coefficients. Most went on to correctly find Q_1

There is still a great deal of confusion in the minds of some candidates over skewness with a number writing reasons such as $Q_1<Q_2<Q_3$. There was a tendency to write wordy explanations rather than the succinct $Q_3 – Q_2>Q_2 – Q_1$. This gained the marks but many candidates were unable to express themselves clearly.

There is still confusion between positive and negative skewness with a few candidates doing correct calculations but concluding it was negative.

A few candidates calculated the mean or mode and used mean > median > mode. These gained full marks if correctly found but used precious time doing unnecessary calculations.

8. A minority of candidates achieved a high rate of success on this question. Part (a) was often badly 1done with lots of fiddling of figures involved in getting to $\frac{1}{5}$.The work was often very poorly organised with lots of crossing out so that candidates did not really know what they had got. Part (b) was often at least half correct. Quite a few candidates stopped at 1.24 but many managed to gain the correct answer. Part (c) was completed correctly by a minority of candidates. Many candidates did not put in limits although they subsequently used them. An incorrect answer of $\frac{1}{20}x^4 + \frac{1}{20}$ was common and $\frac{1}{20}x^4 - \frac{1}{5}$ cropped up in a several cases showing an inability to deal with fractions correctly. Part (d) was answered well by the more able but many candidates equated $\frac{1}{4}x^2$ to a half rather than $\frac{1}{20}x^4 + \frac{1}{5}$.
In part (e) if a candidate attempted this part they generally got 1 mark and often 2 marks.

9. The majority of candidates were able to attempt this question with a high degree of success

(a) Many candidates had a number of attempts at this part before getting a solution. In some cases, responses showed a lack of understanding between the p.d.f. and the c.d.f. This occurred when the candidate differentiated the given function then proceeded to integrate it. The most common error was to interpret the given function as the p.d.f., integrate it and put the answer equal to 1. A small number of candidates took the value of $k = 1/18$ and used it to work backwards.

(b) The most common errors were to find F(1.5) or integrate the given function.

(c) There were many correct solutions with a minority of candidates being unsuccessful. Marks were mainly lost through, having differentiated correctly to find the function, not specifying the p.d.f. fully. A few candidates tried integrating to find the p.d.f.

10. The majority of candidates attempted this question.

(a) Most sketches were clearly labelled with a few omitting the value on the y-axis. Candidates should draw their sketch in the space in the question book, not on graph paper.

(b) A few gave the mode as 2 or 1.

(c) Most were able to find E(X) with only occasional errors in using xf(x) = $2x – 4$ or in substituting the limits.

(d) Finding the median proved challenging for a sizeable minority. Although most wrote that F(m) = 0.5, finding F(m) proved difficult. There were many exemplary solutions but those candidates who struggled got $x^2 – 4x$, but then failed to use the limits correctly or made arithmetic mistakes. It was common for those who had no real understanding to put $2x – 4 = 0.5$ and solve to get $x = 2.25$.

(e) In many cases the answers to this part reflected confusion in understanding the concept of skewness. In many cases where responses were incorrect there was little or no evidence of using the results found, or positive skewness was stated but the reason related to negative skewness.

11. This question has been a good discriminator. The majority of candidates attempted this question with varying degrees of success. In part (a) there were many good sketches with clear labelling but many lost a mark through not marking the patios clearly. In part (b) the most common mistake is to give the value of f(x) i.e. ½. Part (c) was a problem for a majority of candidates. It was evident that many candidates were not competent in finding the CDF of a function given in two parts. Finding F(x) for $0 \le x \le 3$ was reasonably well answered, but quite often candidates did not use the limits or simply wrote down the answers without showing any working. Candidates were less successful in finding F(x) for $3 \le x \le 4$, with few using limits correctly and many not taking into consideration the answer to the first part. Candidates who used the alternative method, using '+c' and F(0) = 0 and F(4) = 1 were generally more successful in getting the correct F(x). Responses to part (d) would seem to reflect a lack of understanding of what the median is. Candidates quoted F(x) = 0.5 and the proceeded to put F(x) for $3 \le x \le 4$ = 0.5 and solve. It was rare to find evidence of candidates checking which part to use before setting up an equation. Many candidates solved F(x) = 0.5 for both parts and then not said which answer was the median. Another common error was adding F(x) for $0 \le x \le 3$ and F(x) for $3 \le x \le 4$ and then solving.

12. The first two parts of this question caused more difficulties for candidates than the later parts. Standard calculations for a uniform distribution are well understood but applying them to a problem caused difficulties for weaker candidates. Stronger candidates had little problem with part (a) but others failed to give a full statement, missing either the values outside the range of the interval or the ranges for the different parts of the density function. In part (b) only better candidates were able to state correctly, and then solve, the two simultaneous equations. In the final part many candidates found $P(X < 30) = 0.2$ but then failed to double this. A common alternative solution was to use the interval $0 < x < 75$.

13. There has been a steady improvement over the years in candidates approach to the questions using given distributions. Weaker candidates are still prone to confusion, but many are able to identify and use the formulas for mean, median and mode correctly. In part (a) most candidates attempted to substitute 0.3 into the given cumulative distribution function but some did not take their answer from 1 to achieve the correct solution. Many candidates substituted the 2 given values in part (b) correctly but not all explained fully why this demonstrated that the median lay between them. When a solution is suggested, then care should be taken that an adequate clear explanation is given. The correct derivative was stated by many candidates in part (c) but some failed to give a full statement of the distribution, missing either the limits or the regions outside the given interval. Integrating $x\mathrm{f}(x)$ correctly and using the limits caused problems for weaker candidates in part (d). A small but appreciable number also made the statement that 16/12 – 9/12 = 5/12. Not all those who differentiated the probability density function placed the differential equal to 0 to obtain the mode. Those who did so usually attained the correct solution. Nearly all candidates who attempted the final part of the question compared at least 2 of the mean, median and mode. The majority who had calculated these correctly were able to identify the skew as negative.

14. This question was well answered by a high percentage of students who gained full marks or only dropped up to four marks. The vast majority of candidates attempted all parts of this question. Evidence of its challenging nature to a number of candidates was the large amount of crossings out and untidy working. This said, however, it was clear candidates had been well prepared for a question of this type. Part (a)was done well with very few “fudged” solutions seen. Most candidates scored full marks. Part (b) was problematic for a number of candidates who simply wrote an incorrect answer without any working, hence losing up to four marks. Others integrated $\mathrm{f}(x)$ but without a variable upper limit or lower limit of 1. A minority of candidates had difficulty with integration. A significant number of candidates lost many marks on this answer through using k instead of $1/k$ in their working for parts (b), (c), and (d). Candidates who lost marks on part (b) often gained marks later for parts (c) and (d) through working from the original function rather than using their answer to part (b). In part (c) most candidates knew how to find the mean, although a few tried to integrate $x\mathrm{F}(x)$ rather than $x\mathrm{f}(x)$. In part (d) many candidates knew what to do to find the median with the majority of marks lost because the wrong expression for $\mathrm{F}(X)$ was used. A few poor solutions of quadratic solutions were seen but it was good to see many candidates correctly discard the unwanted solution. In part (e) many candidates differentiated to find the mode which was inappropriate in this case. However quite a number drew a good sketch and used this to correctly identify the mode. In part (f) the inequality mean<median<mode was generally known and quoted, often in spite of conflict with their answers to the previous parts!

15. There is an increasing number of candidates who are able to attempt questions of this type successfully. However, many still have difficulty with the calculus required and/or calculating and using the cumulative distribution function.

Part (a) was completed successfully by many candidates who showed clearly how the given solution was achieved.

In part (b) only the weaker candidates had problems in calculating the integral required.

In part (c) the question was completed entirely successfully by a minority of candidates. The most common error was the failure to use an upper variable limit and a lower limit of 2 for the integration. Those using the equivalence method to find the constant of integration were usually successful. A few candidates failed to show the full distribution function F(x) omitting $x \leq 2$ and $x \geq 3$.

Some candidates did some quite remarkable lengthy, but incorrect, algebraic work on the cumulative distribution function in part (d). Generally however candidates did manage to substitute but a number forgot to say that F (x) = ½.

16. Most candidates responded well to this question. However, a smallish number were rather confused; integrating instead of differentiating, and *vice versa*, confusing mode and median and substituting $x = 0.5$ in $F(x)$ instead of solving $F(x) = 0.5$ in part (d). The majority of candidates provided satisfactory solutions to part (a) and earned all four marks, although a very small number attempted to 'fake' the proof. The most common error was the omission of "= 1". There were a large number of perfect solutions to part (b) with concise and accurate working. The overall response to part (c) was again satisfactory, although a sizeable minority used f(x) or integrated. In part (d) a majority of candidates established the quartic equation. However, this was as far as most of them went. A small proportion went on to solve the disguised quadratic, usually using 'the formula', although 'completing the square' was seen on a few occasions. A minority attempted numerical techniques to obtain an answer correct to three significant figures. All too many candidates produced this incorrect solution;

$$x^2(8 - x^2) = 8$$

$$x^2 = 8 \text{ or } 8 - x^2 = 8$$

$$x = \sqrt{8} \text{ or } x = 0$$

In part(e) the appropriate comparisons were generally made although this did not always lead to the correct conclusions being made. There were many correct answers, although if the evidence of the responses from parts (e) and part (f) were considered together, it could be construed that some candidates were guessing. The first mark in part (f) was mostly for administration; labels and the horizontal sections. The lines x<0 and x>2 were often omitted and labels were often incorrect. For the second mark, too many candidates drew small, inaccurate sketches that were symmetrical with the mode at x=1 rather than slight negative skew.

17. Many correct solutions to this question were seen, but there were also some poor solutions resulting from untidy working and poor arithmetic when substituting limits. The candidates seemed to know what methods to use but they could not always apply them accurately. Their integration and differentiation techniques were fine but using them in the various parts was at times disappointing. More care and attention to detail was needed.

18. This proved to be a good discriminator with a few candidates scoring full marks with ease, whilst others struggled with some or all of the four parts. Part (a) proved to be the most accessible part with many correct solutions seen by the examiners. Common errors included working out $\int x\mathrm{f}(x)\,\mathrm{d}x$ for only one part of their probability density function or adding up the two correct definite integrals and dividing their answer by two. Part (b) was the most challenging part of this question. Many candidates were able to find F(x) between x = 0 and x =1, but did not realise that when finding F(x) from x = 1 to x = 2, they had to include the probability (or area) up to x=1. Many candidates were able to specify F(x) for x < 0 and x > 2. In part (c), some candidates realised that they needed to solve the equation F(m) = 0.5, by using their cumulative distribution function in part (b). Common errors included finding F(0.5); differentiating their F(x) and setting the result equal to zero; solving f(x) = 0; or solving F(m) = 0.5 using their part of the CDF for $0 \le x < 1$. In part (d), some candidates were able to justify that the distribution was negatively skewed by making reference to the mean being less than the median.

19. Nearly all candidates achieved some marks for this question but few gave completely correct solutions. In part (a) most candidates showed their working clearly and attempted to integrate f(x), although some made errors when performing this integration. The lack of detail in the integration in part (b) cost many candidates the first two marks, but most managed to score something in this part, even if only the mark for the 'ends' of the distribution. Fewer than would have been expected appreciated that all that was required was to solve $\mathrm{f}'(x) = 0$ in part (d).

A common error was to solve $\mathrm{F}'(x)=0$ instead. Only the best candidates realised what was required in part (e) with a number of candidates attempting to solve an equation with little success. In part (f) candidates failed to realise that by using the solutions to the previous three parts of the question they could arrive at the correct answer. Few candidates were able to state the inequalities correctly and interpret them and score both marks.

20. This question was popular with the candidates. In part (a) there were many fully correct solutions but some candidates did not deal with the limit of zero and fudged their answer at the end to get positive 3. In part (b), again, many candidates did not substitute in the limit of zero and ended up with 0.25 rather than – 0.25. In part(c) many candidates did not realize that they needed to integrate the function using a lower limit of –1. In part (d) a large proportion of candidates were caught out when evaluating F(0.3). Many candidates just substituted 0.3 into their cumulative distribution function not realizing that it was only valid for $-1 \le X \le 0$.

21. There was a sizeable group of candidates who answered this question well as they were able to demonstrate a good grasp of the mathematical technique required. Equally well there were other candidates who had a better grasp of statistical reasoning, but for whom integration was something whose relevance to statistics was yet to become apparent. Most candidates did not approximate the answer in part (a) appropriately. There were very many completely correct solutions to parts (b) and (c) and remarkably little evidence of fiddling to obtain the given variance. Part (d) proved the most difficult part and only the better candidates realised the need for differentiation to find the mode, a significant minority relying on dubious sketches instead.

22. No Report available for this question.

23. No Report available for this question.